SHEAR AND VORTICITY IN AN ACCELERATING BRANS-DICKE LAMBDA-UNIVERSE WITH TORSION

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Abstract

We study accelerating Universes with power-law scale-factors. We include shear and vorticity, a cosmological "constant" term, and spin from torsion, as in Einstein-Cartan's theory when a scalar-field of Brans-Dicke type acts in the model. We find a "no-hair" result, for shear and vorticity; we also make contact with the alternative Machian picture of the Universe.

Keywords: Cosmology; Einstein; Brans-Dicke; Cosmological term; Shear; Spin; Vorticity; Inflation; Einstein-Cartan; Torsion; Accelerating Universe.

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I. Introduction

The existence of shear (σ), vorticity (ϖ) and cosmological "constant" (Λ), have not been well discussed in the context of a scalar field theory, say, compatible with Brans-Dicke (1961) theory, with or without torsion, for an accelerating present Universe. The purpose of this paper is two-fold:

i) to show that there is a "no-hair" power-law result, under which shear and vorticity are erased from the model as time passes by; and to describe a valid model.

ii) to show that when the space is torsioned by a spin of the Universe, the most obvious model, results in the same kind of time-evolution for the spin as in the Machian Universe, as described by Berman(2007; 2007a, b, c; 2008; 2008a, b), thus pointing to the rotation, plus expansion, state.

Prior work by Berman (references above) on an inflationary solution for the exponential expansion of the same kind of lambda-Universe, resulted in the same conclusions.

We first analyse a Brans-Dicke scalar-field (Section II and III), and later treat the inclusion of torsion *à la* Einstein-Cartan, while keeping the scalar-field.

II. Brans-Dicke model with shear and vorticity

From the scalar-tensor cosmologies, the Brans-Dicke theory renders the most simple case (Brans and Dicke, 1961). For details on this, and other scalar-tensor theories, we advise the reader to consult the books by Berman (2007a), Faraoni (2004) and Fujii and Maeda (2003).

By the same procedure as in the inflationary case, we take Raychaudhuri's equation for this case, in *Einstein's frame*, which mimicks Einstein's theory in non-conventional units (Dicke, 1962), whereby the scale-factor and time are scaled by $\phi^{-\frac{1}{2}}$ while mass goes with $\phi^{\frac{1}{2}}$. Afterwards, we shall proceed to the conversion to the *Jordan-frame*, i.e., in the conventional set of units.

Consider now Robertson-Walker's metric,

$$ds^2 = dt^2 - \frac{R^2(t)}{\left[1+\left(\frac{kr^2}{4}\right)\right]^2} d\sigma^2 \quad , \tag{1}$$

where $k = 0$ and $d\sigma^2 = dx^2 + dy^2 + dz^2$.

When we include a cosmological "constant" term, the field equations in the *Einstein's frame* read, for a perfect fluid,

$$\frac{8\pi G}{3}\left(\rho + \frac{\Lambda}{\kappa} + \rho_\lambda\right) = H^2 \equiv \left(\frac{\dot{R}}{R}\right)^2 \quad . \tag{2}$$

$$-8\pi G\left(p - \frac{\Lambda}{\kappa} + \rho_\lambda\right) = H^2 + \frac{2\ddot{R}}{R} \quad . \tag{3}$$

In the above, we have:

$$\rho_\lambda = \frac{2\omega+3}{32\pi G}\left(\frac{\dot{\phi}}{\phi}\right)^2 = \rho_{\lambda 0}\left(\frac{\dot{\phi}}{\phi}\right)^2 \quad . \tag{4}$$

The complete set of field equations, must be complemented by three more equations, of which only two are independent, when taken along with (2),(3) and (4): one is the dynamical fluid equation; the other is the continuity one, as follow:

$$\frac{d}{dt}\left[\left(\rho + \rho_\lambda + \frac{\Lambda}{\kappa}\right) R^3\right] + 3R^2\dot{R}\left[p - \frac{\Lambda}{\kappa} + \rho_\lambda\right] = 0 \quad , \tag{5}$$

and,

$$\frac{d}{dt}\left[\left(\rho + \frac{\Lambda}{\kappa}\right) R^3\right] + 3R^2\dot{R}\left[p - \frac{\Lambda}{\kappa}\right] + \frac{1}{2}R^3\frac{\dot{\phi}}{\phi}\left[\rho + \frac{4\Lambda}{\kappa} - 3p\right] = 0 \quad . \tag{6}$$

Raychaudhuri's equation (Raychaudhuri, 1979), as derived by Berman, becomes, when we include shear and vorticity,

$$3\dot{H} + 3H^2 = \dot{u}^\alpha_{;\alpha} + 2\left(\varpi^2 - \sigma^2\right) - 4\pi G\left(\rho + 3p + 4\rho_\lambda\right) + \Lambda \quad , \tag{7}$$

where, H and $\dot{u}^\alpha_{;\alpha}$ stand for Hubble's parameter, and the acceleration of the fluid, which from now on, will be considered null ($\dot{u}^\alpha_{;\alpha} = 0$).

We remember that we first deal with a kind of Einstein's cosmology, so that Λ stands constant and we make later the transformation towards conventional units or, Jordan's frame. In the next Section, we present a power-law model that encompasses the possible accelerating Universe, i.e., where the deceleration parameter is negative ($q < 0$).

III. Accelerating power-law BD-Universe

Consider power-law scale-factors, with constant deceleration parameter, like in the original papers by Berman (1983) and Berman and Gomide (1988).

$$R = (mDt)^{\frac{1}{m}} \quad , \quad (\; D = \text{ constant }) \tag{8}$$

where,

$$m = q + 1 = \text{ constant} \quad . \tag{9}$$

We now find the following solution,

$$\begin{aligned} H &= (mt)^{-1} \quad ; \\ \Lambda &= \Lambda_0 t^{-2} \quad ; \\ \varpi^2 &= \varpi_0^2 t^{\gamma} \quad ; \\ \sigma^2 &= \sigma_0^2 t^{\gamma} \quad ; \\ \rho &= \rho_0 t^{\beta} \quad ; \\ p &= p_0 t^{\beta} \quad ; \\ \rho_\lambda &= \delta^2 \rho_{\lambda 0} t^{-2} \quad ; \end{aligned} \tag{10}$$

and,

$$\phi = \phi_0 t^{\delta} \quad ,$$

where, Λ_0 , ϖ_0^2 , σ_0^2 , ρ_0 , p_0 , $\rho_{\lambda 0}$, β , γ , δ and ϕ_0 are constants.

In order that the model be consistent, we need to impose the following conditions,

$$\beta = \gamma = -2 \quad , \tag{11}$$

$$\frac{3}{m}\left(\frac{1}{m} - 1\right) = 2\left(\varpi_0^2 - \sigma_0^2\right) - 4\pi\left(\rho_0 + 3p_0 + 4\delta^2\rho_{\lambda 0}\right) + \Lambda_0 \quad . \tag{12}$$

When we go back to Jordan's frame, we find:

$$\begin{aligned}
\bar{R} &= R\phi^{\frac{1}{2}} \quad ; \\
\bar{\rho} &= \rho\phi^{-2} \quad ; \\
\bar{p} &= p\phi^{-2} = \left[\frac{p_0}{\rho_0}\right]\bar{\rho} \quad ; \\
\bar{\varpi} &= \varpi\phi^{-\frac{1}{2}} \quad ; \\
\bar{\sigma} &= \sigma\phi^{-\frac{1}{2}} \quad ; \\
\bar{\Lambda} &= \Lambda_0\phi^{-1} \quad ; \\
\bar{\phi} &= \phi \quad ; \\
\bar{H} &= H\phi^{-\frac{1}{2}} \quad .
\end{aligned} \tag{13}$$

When we plug (10) and (11) in (13), we obtain,

$$\begin{aligned}
\bar{R} &= \phi_0^{\frac{1}{2}}\left(mD\right)^{\frac{1}{m}} t^{\left(\frac{\delta}{2} + \frac{1}{m}\right)} \quad ; \\
\bar{\rho} &= \rho_0\phi_0^{-2} t^{-2(\delta+1)} \quad ; \\
\bar{p} &= p_0\phi_0^{-2} t^{-2(\delta+1)} \quad ; \\
\bar{\varpi} &= \varpi_0\phi_0^{-\frac{1}{2}} t^{-\left(1+\frac{\delta}{2}\right)} \quad ; \\
\bar{\sigma} &= \sigma_0\phi_0^{-\frac{1}{2}} t^{-\left(1+\frac{\delta}{2}\right)} \quad ; \\
\bar{\Lambda} &= \Lambda_0\phi_0^{-1} t^{-(\delta+2)} \quad ;
\end{aligned} \tag{14}$$

$$\bar{H} = m^{-1}\phi_0^{-\frac{1}{2}} t^{-\left(1+\frac{\delta}{2}\right)} \quad .$$

It can be checked that we have the "no-hair" condition, for an accelerating Universe, with $\delta > -1$ and also $\delta > -\frac{2}{m}$, the scale-factor increases up to infinity when we shall find that Hubble's parameter $\bar{H} \longrightarrow 0$. The condition $\delta > -2$ makes shear and vorticity disappear with growing time.

IV. Scalar-field with torsion in an accelerating Universe

We refer to Einstein-Cartan's theory (Cartan, 1923). It was shown by Berman (2008a), that the same model used in obtaining inflationary torsioned solutions, can be retrieved from the Brans-Dicke case, with shear, vorticity and lambda-term, by including a spin density, S , and a total universal spin S_U , where,

$$S_U = SR^3 \quad . \tag{15}$$

We refer to this last reference for the passage from Einstein's to Jordan's frame, for spin, and for the combined field equations with torsion, which does not affect its formula, i.e.,

$$\bar{S}_U = S_U \quad . \tag{16}$$

It must be clear that the spin density term must be included in the resulting Raychaudhuri's equation, whereby, from the relation (7) we obtain the new one by making the substitution below,

$$\Lambda \longrightarrow [\Lambda + 128\pi^2 S^2] \quad . \tag{16a}$$

We can check, that in the given model for the BD case, we need to include an Einstein-Cartan's theory with,

$$S^2 = s_0^2 t^\gamma \quad , \tag{17}$$

where γ has the value of Section III ($\gamma = -2$).

Therefore, we obtain,

$$\bar{S}_U = s_0 \, (mD)^{\frac{3}{m}} \, t^{\frac{3}{m}-1} \qquad . \tag{18}$$

This closes the calculation.

V. Conclusions

The "no-hair" property of the otherwise perfect fluid, has been proved for the present model. As to the spin of the Universe, we have two comments:

– **FIRST**) the form of the spin is of the type,

$$\bar{S}_U \propto R^3 t^{-1} \qquad .$$

In the Machian limit, the scale-factor is proportional to the age of the Universe. Then, the Machian contact with the above formula, is fulfilled by considering,

$$\bar{S}_U \propto R^2 \qquad .$$

The papers by Berman cited above, contain this Machian conclusion.

– **SECOND**) though the spin grows with time, Berman (2008; 2008a; 2008b) has shown that the angular speed decreases with R^{-1} (de Sabbata and Gasperini, 1985).

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